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ler: LVK (LIGO-Virgo-KAGRA collaboration) event (compact-binary mergers) rate calculator and simulator

Hemantakumar Phurailatpam [1], **Anupreeta More** [2,3], **Harsh Narola** [4,5], **Leo C. Y. Ng** [1,6], **Justin Janquart** [4,5,7,8], **Chris Van Den Broeck** [4,5], **Otto Akseli Hannuksela** [1], **Neha Singh** [9], and **David Keitel** [9]

1 Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong **2** The Inter-University Centre for Astronomy and Astrophysics (IUCAA), Post Bag 4, Ganeshkhind, Pune 411007, India **3** Kavli Institute for the Physics and Mathematics of the Universe (IPMU), 5-1-5 Kashiwanoha, Kashiwa-shi, Chiba 277-8583, Japan **4** Department of Physics, Utrecht University, Princetonplein 1, 3584 CC Utrecht, The Netherlands **5** Nikhef – National Institute for Subatomic Physics, Science Park, 1098 XG Amsterdam, The Netherlands **6** IGC, Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, PA 16802, USA **7** Center for Cosmology, Particle Physics and Phenomenology - CP3, Universit'e Catholique de Louvain, Louvain-La-Neuve, B-1348, Belgium **8** Royal Observatory of Belgium, Avenue Circulaire, 3, 1180 Uccle, Belgium **9** IAC3, Universitat de les Illes Balears, Crta. Valldemossa km 7.5, E-07122 Palma, Spain

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Software

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- @sibirrer
- @Jammy2211

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Summary

Gravitational waves (GWs) are ripples in spacetime produced by accelerating massive objects. Since 2015, the LIGO-Virgo-KAGRA (LVK) collaboration (Acernese et al., 2014; Akutsu et al., 2020; The LIGO Scientific Collaboration et al., 2015) has routinely detected GW transients from compact-binary mergers, including binary black holes (BBHs), binary neutron stars (BNSs), and neutron star-black hole pairs (NSBHs), during observing runs (R. Abbott et al., 2023; The LIGO Scientific Collaboration, the Virgo Collaboration, and the KAGRA Collaboration, 2026; R. Abbott et al. 2021, *PRX*; B. P. Abbott et al., 2019). Most of these signals reach detectors unobstructed, forming a baseline unlensed population, but a small fraction can encounter massive intervening objects such as galaxies or galaxy clusters. These objects act as lenses, splitting the signals into multiple images that are magnified, time-shifted, and phase-shifted through strong gravitational lensing (R. Abbott et al. 2021, *ApJ*; Wierda et al., 2021).

Accurate modeling of unlensed and lensed gravitational-wave populations is essential for astrophysical interpretation, inferring merger rate histories, and validating future lensing events (R. Abbott et al. 2021, *ApJ*; Janquart et al., 2023; The LIGO Scientific Collaboration, the Virgo Collaboration, and the KAGRA Collaboration, 2024; Wierda et al., 2021). The `ler` software is a Python framework that provides a unified pipeline to model these populations statistically and calculate their detectable rates. Producing precise results requires computationally demanding tasks such as large-scale sampling of source and lens attributes, solving lens equations, and simulating detector responses. `ler` reduces these costs through optimization and parallelization, achieving up to a thousandfold speedup relative to an un-optimized baseline implementation (further details provided in the Software design section). Source code, validation examples, and documentation are available through the project repository, supporting examples, and documentation site.

Statement of need

`ler` is a statistics-based Python tool designed for the joint simulation and computation of detectable rates for both unlensed and lensed gravitational-wave events. It is intended for LVK analyses and for astrophysics researchers working on compact-binary populations and strong lensing. Its core functionality integrates the sampling of compact-binary source properties and lens-galaxy characteristics, solving lens equations to derive image properties, and computing detectable event rates. This functionality leverages NumPy (Harris et al., 2020) for array operations and linear algebra alongside SciPy (Virtanen et al., 2020) interpolation methods and Python multiprocessing. Efficiency is improved using Numba (Lam et al., 2015) for just-in-time compilation and multithreading, and by using inverse-transform sampling and importance sampling where rejection sampling is inefficient. `ler` can use `lenstronomy` (Birrer et al., 2021) for lensing calculations and `gwsnr` (Phurailatpam & Hannuksela, 2025) for efficient detection-probability calculations $P_{\rm det}$.

`ler` produces parameter distributions for intrinsic populations and for populations selected by detection criteria, including strong-lensing selection and image-level detectability. This supports validation studies for lensing candidates (Janquart et al., 2023) and forecasting for present and future detector networks (Ng et al., 2025). The design can be adapted to multi-messenger applications by changing the detection criteria, including studies of strongly lensed gamma-ray bursts (More & Phurailatpam, 2025). `ler` also supports integration with existing inference tools by providing $P_{\rm det}$ estimates that can be used to construct selection functions (see supporting examples) for hierarchical population inference frameworks (Thrane & Talbot, 2019) with Bilby software (Ashton et al., 2019).

State of the field

Joint population studies of unlensed and strongly lensed gravitational waves require end-to-end simulations that combine source and lens-galaxy models with detector selection effects. Common inference packages such as Bilby (Ashton et al., 2019) and cosmology and rate tools such as `gwcosmo` (Gray et al., 2020, 2022, 2023) provide general-purpose likelihood evaluation, sampling, and population inference utilities. However, they do not provide a single, configuration-driven pipeline to generate unlensed and strongly lensed populations jointly at scale, nor to propagate lensing image properties into detectability. Lens-modelling tools such as `lenstronomy` (Birrer et al., 2021) implement fast analytical routines for lensing calculations but are not designed as high-throughput population-level simulators.

Previous strong-lensing population studies often relied on pipelines tightly coupled to specific lens models or detection criteria (Haris et al., 2018; Wierda et al., 2021). Many studies report rate estimates based on limited sample sizes without showing convergence diagnostics or uncertainty estimates. They also commonly neglect uncertainty in population-model hyperparameters, which should propagate into the detectable population and rate predictions. `ler` addresses these gaps by providing an integrated, high-throughput workflow linking population draws, lensing cross-sections, image properties, and detector selection in one reproducible pipeline.

Software design

`ler` is designed as an end-to-end population simulator with a clear separation between the scientific steps and the computational machinery. The pipeline is organized around two rate calculations. Unlensed rates are computed by drawing compact-binary populations from astrophysical priors, evaluating detectability, and integrating over the detected events. Lensed rates extend this flow by sampling lens-galaxy and source-redshift parameters under a strong-lensing condition, solving the lens equation to generate image properties, applying detectability criteria at the image level, and integrating over the detectable lensed events. This separation keeps the rate logic explicit while allowing the source, lens, and detection models to be replaced without rewriting the full pipeline. The mathematical foundations for these implementations are detailed in the Mathematics section.

A core design choice is modularity, with stable interfaces between modules that handle source populations, lens populations, image-property calculations, and rate estimation. Users can select built-in models such as BBH, BNS, or NSBH populations and lens models such as singular isothermal sphere (SIS), singular isothermal ellipsoid (SIE), or elliptical power-law with external shear (EPL+Shear), or provide their own distributions and detection criteria. Detector selection is handled through a backend interface so that signal-to-noise and detection probability calculations can be delegated to `gwsnr` or replaced by alternative implementations. This architecture supports extension and testing while keeping the scientific assumptions visible in configuration rather than embedded in ad hoc scripts.

Performance is obtained by compiling the repeatedly evaluated parts of the pipeline with Numba `njit` and parallelizing the main Monte Carlo loops. `ler` uses `lenstronomy` (Birrer et al., 2021) as an optional backend, but it provides an in-house analytical EPL+Shear solver and caustic and cross-section calculator that are compiled with Numba and run in parallel, following the same mathematical formulation as `lenstronomy` and Tessore & Benton Metcalf (2015). This leads to two practical trade-offs. First, the rate estimates are Monte Carlo approximations, so accuracy is controlled by sample size rather than closed-form expressions, and `ler` reports convergence statistics from repeated batches. Second, just-in-time compilation adds overhead on first use, which is about 2 s for the first unlensed batch and about 25 s for the first lensed batch that includes image-property calculations. After compilation, the compiled kernels reduce per-sample cost and make large simulations feasible. In benchmarks that disable just-in-time compilation and parallelism, these choices provide speed gains of about $180\times$ for unlensed populations and $5189\times$ for lensed populations. On a standard six-core processor (Apple M2 Pro, 16GB RAM), a batch of 10^5 samples takes about 300 ms for unlensed calculations and about 30 s for lensed calculations, and repeated batches converge to a stable rate estimate.

Mathematics

The mathematical workflow in `ler` can be summarised as Monte Carlo estimates of detectable event rates for unlensed and strongly lensed populations, obtained by averaging detection probabilities over simulated events drawn from the relevant population models. For unlensed events, the detector-frame rate of detectable events is

$$\frac{\Delta N_{\rm U}^{\rm obs}}{\Delta t} = \mathcal{N}_{\rm U} \left\langle P({\rm obs} \mid \vec{\theta}) \right\rangle_{\vec{\theta}\sim P(\vec{\theta})}$$

where $\mathcal{N}_{\rm U}$ is the total intrinsic merger rate in the detector frame, $\vec{\theta}$ denotes the source parameters drawn from $P(\vec{\theta})$, and $P({\rm obs} \mid \vec{\theta})$ is the detection probability.

For strongly lensed events, the detector-frame rate of detectable lensed events is

$$\frac{\Delta N_{\rm L}^{\rm obs}}{\Delta t} = \mathcal{N}_{\rm L} \left\langle P({\rm obs} \mid \vec{\theta}_{\rm U}, \vec{\theta}_{\rm L}, \vec{\beta}, {\rm SL}) \right\rangle_{\substack{\vec{\theta}_{\rm U},\vec{\theta}_{\rm L}\sim P(\vec{\theta}_{\rm U},\vec{\theta}_{\rm L}|z_L,z_s,{\rm SL}) \\ \vec{\beta}\sim P(\vec{\beta}|z_s,\vec{\theta}_{\rm L},{\rm SL})}},$$

where $\mathcal{N}_{\rm L}$ is the total intrinsic merger rate in the detector frame for the lensed population, $\vec{\theta}_{\rm U}$ and $\vec{\theta}_{\rm L}$ denote the source and lens parameters, $\vec{\beta}$ is the source position in the source plane, and z_L and z_s are the lens and source redshifts, respectively. The distributions

$P(\vec{\theta}_{\rm U}, \vec{\theta}_{\rm L} \mid z_L, z_s, {\rm SL})$ and $P(\vec{\beta} \mid z_s, \vec{\theta}_{\rm L}, {\rm SL})$ define the hierarchical sampling under the strong-lensing condition SL, and $P({\rm obs} \mid \vec{\theta}_{\rm U}, \vec{\theta}_{\rm L}, \vec{\beta}, {\rm SL})$ is evaluated using the image properties implied by $(\vec{\theta}_{\rm L}, \vec{\beta})$ and the chosen detectability criteria.

Research impact statement

`ler` has established a measurable impact within the gravitational-wave community through its integration into research workflows and its growing user base. The package supports a diverse community ranging from students to faculty members at multiple institutions, with development driven by researchers across various organizations and peer-reviewed within the LVK Scientific Collaboration. Active community engagement is maintained through bug reports and feature requests on the GitHub issue tracker and through collaboration communication channels such as LIGO Mattermost.

The software has facilitated several scientific publications related to gravitational-wave lensing statistics. It has been used to analyze the distribution of time delays and magnification ratios for the validation of lensing candidates (Janquart et al., 2023) and to forecast the detection of sub-threshold lensed events with future observatories (Ng et al., 2025). Additional research applications include the generation of lensed-event statistics and the calculation of posterior odds (Hannuksela et al., 2026), as well as forecasting electromagnetic counterparts for strongly lensed gamma-ray bursts (More & Phurailatpam, 2025). These implementations demonstrate that the tool provides a robust framework for high-level population studies and astrophysical forecasting.

AI usage disclosure

No generative AI tools were used to develop the software, write this manuscript, or prepare the accompanying materials.

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