# HW2D: A reference implementation of the Hasegawa-Wakatani model for plasma turbulence in fusion reactors

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Summary

HW2D is an open source reference implementation of the Hasegawa-Wakatani model against which new methods are benchmarked, validated, and trained upon. The model has become the de-facto standard model for testing new methods on turbulence in fusion devices. Besides providing consistent numerical simulations, it implements the physical property calculations with the normalizations common in reference texts that are often not clearly understandable outside of the community. Along with it, an iteratively expanding list of stable parameters are published for simulations with associated physical properties. This drastically reduces the resource usage of grid searches for stable turbulent simulations of a complex system and the time needed to verify implementation for testing new methods.

Statement of need

Nuclear fusion provides the promise of clean, endless energy for humanity in the future. One of the biggest open questions standing in the way of commercially viable fusion devices is confining the energy within the plasma that is transported out through drift wave turbulence. However, the immense complexity of modeling turbulence inside potential reactors still remains outside the realm of the biggest supercomputers. Hence, the development of new methods for simulating plasma turbulence is a pressing issue with the speed at which new methods are being tested expanding rapidly.

The Hasegawa-Wakatani (HW) model has become the de-facto standard for testing methods, especially those developed for Navier-Stokes (NS) fluids, for fusion turbulence due to its similarity to it (Camargo et al., 1995). Although first published 40 years ago (Hasegawa & Wakatani, 1983), it is still missing an open-source reference implementation. With its rising popularity in other fields, especially machine learning to test limits of physics-based learning methods, a simple implementation is needed. A unified reference allows comparing methods (Anderson et al., 2020; Goumiri et al., 2013; Heinonen & Diamond, 2020; Stegmeir et al., 2018; Yatomi et al., 2023) and reduces the overhead involved in verifying turbulent system implementations. This verification is now automated with CI pipelines and open-sourced for the broader community to use.

In summary, HW2D provides an easy to understand, easy to use, and easy to extend reference implementation to help accelerate the testing of new ideas and methods. This stands in contrast to the closed-source Fortran and C++ codes available today at a select few research institutes. As an open-source Python reference solver, HW2D opens up the research field beyond these confines. It significantly decreases the barrier of entry for fusion- and machine learning reserchers to tackle one of the biggest open questions in contemporary science. This implementation was and is being used for scientific papers on data-driven methods for plasma

turbulence and as an introduction to plasma turbulence for new researchers.

Mathematics

The HW model describes drift-wave turbulence using two physical perturbation fields: the density n and the potential ϕ perturbations above the background, with the vorticity defined as $\Omega = \nabla^2 \phi$. Their evolution over time is described by two coupled PDEs:

$$\partial_t n = c_1 \left(\phi - n \right) - [\phi, n] - \kappa_n \partial_y \phi - \nu \nabla^{2N} n \ ,$$

$$\partial_t \Omega = c_1 \left(\phi - n \right) - [\phi, \Omega] - \nu \nabla^{2N} \Omega \ .$$

Reference Implementation

The reference impementation set forth is using numerical simulations in physical space. The methods are second order accurate in space, and fourth order accurate in time. This marks a balance of simplicity and efficiency to allow intrusive methods to be tested easily. However, the use of the Arakawa Scheme for the Poisson brackets does allow the preservation of higher order metrics, while preventing certain artefacting.

- Gradients ∂_x: 2nd order accurate Central Finite Difference scheme
- Poisson Bracket $[\cdot, \cdot]$: Arakawa Scheme (Arakawa, 1997)
- Spatial Derivative $\nabla_\perp$: Laplace as 2D Central Finite Difference
- Poisson Equation $\nabla^{-2}\cdot$: Fourier-based Poisson Solver
- Time Integration ∂_t: 4th order explicit Runge-Kutta scheme

Physical Properties

The reason why the Hasegawa-Wakatani Model has been the de-facto testing bed for new methods are its verifiable statistically stationary properties for the complex turbulent system, such as the turbulent particle flux Γ^n, primary sink Γ^c, energy E, and enstrophy U:

$$\Gamma^n = - \iint \mathrm{d}^2 x \ \left(n \, \partial_y \phi \right)$$

$$\Gamma^c = c_1 \iint \mathrm{d}^2 x \ \left(n - \phi \right)^2$$

$$E = \frac{1}{2} \iint \mathrm{d}^2 x \ \left(n^2 - \left| \nabla_\perp \phi \right|^2 \right)$$

$$U = \frac{1}{2} \iint \mathrm{d}^2 x \ \left(n - \nabla_\perp^2 \phi \right)^2 = \frac{1}{2} \iint \mathrm{d}^2 x \ \left(n - \Omega \right)^2 \ .$$

These can be complemented with spectral definitions of these, or the characteristic phase shift $\delta(k_y)$ between the density n and potential ϕ:

$$\int \mathrm{d}k_y \, \Gamma^n(k_y) \ = - \int \mathrm{d}k_y \ \left(i \, k_y \, n(k_y) \, \phi^*(k_y) \right)$$

$$\delta(k_y) \ = -\mathrm{Im}(\log \left(n^*(k_y) \, \phi(k_y) \right))$$

$$E^N(k_y) \ = \frac{1}{2} \left| n(k_y) \right|^2$$

$$E^V(k_y) \ = \frac{1}{2} \left| k_y \, \phi(k_y) \right|^2 \ .$$

Beside the spectral verification, time-integration can be verified through the expected in- and out-flows in the turbulent phase:

$$\partial_t E = \Gamma^N - \Gamma^c - \mathfrak{D}^E$$
$$\partial_t U = \Gamma^N - \mathfrak{D}^U \,.$$

Where $\mathfrak{D}^E$ and $\mathfrak{D}^U$ are sinks derived from the diffusion terms of the HW equations, namely:

$$\mathfrak{D}^E = \iint \mathrm{d}^2x \, (n\,\mathfrak{D}^{\mathfrak{n}} - \phi\,\mathfrak{D}^{\phi})$$

$$\mathfrak{D}^U = -\iint \mathrm{d}^2x \, (n - \Omega)(\mathfrak{D}^n - \mathfrak{D}^{\phi})$$

$$with \quad \mathfrak{D}^n(x,y) = \nu\,\nabla^{2N} n \quad and \quad \mathfrak{D}^{\phi}(x,y) = \nu\,\nabla^{2N}\phi \,.$$

Note that it is the common practice across all reference texts to calculate the integral $\int \cdot$ as the average over a unit square $\langle \cdot \rangle$ in order to get comparable values for all properties.

The model code includes the most important properties and their very specific normalizations required to compare against literature values. It goes further, however, and provides its own reference values with—for the first time—statistical bounds for a vast range of values. This allows simple comparison, as well es evalutaion of new methods to one reference community built resource.

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