pySBeLT: A Python software package for stochastic sediment transport under rarefied conditions

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Summary

Granular sediment of various sizes moves downstream along river beds when water flow is capable of entraining particles from the bed surface. This process is known as bed load sediment transport because the particles travel close to the boundary. It is common to treat the transport process as a predictive problem in which the mean transport rate past a stationary observation point is a function of local water flow conditions and the grain size distribution of the bed material (Ancey, 2020; Parker, 2008; Wainwright et al., 2015). However, a predictive approach to the bed load problem neglects the stochastic nature of transport due to the movements of individual particles (Einstein, 1937; Furbish & Doane, 2021), and interactions between moving particles and those on the bed surface (Ancey et al., 2006, 2008; Lee & Jerolmack, 2018). Here, we present an open-source Python model, pySBeLT, which simulates the kinematics of rarefied particle transport (low rates) as a stochastic process along a riverbed profile. pySBeLT is short for *Stochastic Bed Load Transport*. The primary aim of pySBeLT is to offer an efficient and reasonable numerical means to probe connections between individual particle motions and local transport rates, or the flux. We suggest that pySBeLT is a suitable teaching tool to help introduce bed load transport to advanced undergraduate and graduate students alike.

Statement of need

Research at the intersection of geomorphology, geophysics and hydraulics is increasingly focused on building a theoretical foundation for the treatment of bed load transport as a stochastic phenomenon (Ancey, 2020; Furbish & Doane, 2021). Associated theories are commonly tested against laboratory data from "rarefied" transport conditions (Furbish et al., 2016), where transport rates are low to moderate, interactions between two or more moving particles are rare, and a relatively small fraction of particles on the bed surface participate in transport (Ancey, 2010; Fathel et al., 2015; Roseberry et al., 2012; Wu et al., 2020). For example, laboratory experiments using a downstream light table counting device and conducted at roughly twice the shear stress threshold for particle motion involve the transport of less than approximately 12% of particles on the upstream bed surface (Chartrand, 2017). This result suggests that the flux measured across a boundary or within an area of bed surface is directly linked to the motions of individual particles arriving from upstream locations (Furbish et al., 2012).

Particle motions are controlled by several influencing factors including fluid turbulence, the irregular bed surface, and collective effects (Ancey et al., 2006, 2008; Lee & Jerolmack, 2018). As a result, the connection between particle movements and the bed load transport rate has been difficult to formulate mathematically. pySBeLT provides an extensible framework

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within Python to numerically examine correlations between upstream particle entrainment rates and travel distances, with downstream flux. pySBeLT was motivated by a birth-death, immigration-emigration Markov model for bed load transport (Ancey, 2010; Ancey et al., 2008). Here, the movements of individual particles are represented by stochastic entrainment, motion, and deposition processes, and sediment flux is represented as a counting phenomenon where the number of particles in motion above the bed surface is a random variable (Ancey et al., 2008). The model supports ensemble simulations so that repeat numerical experiments can be conducted efficiently, or the problem can be probed across a range of input parameter values (discussed below).

pySBeLT is run forward in time according to default or user specified parameter values. After initialization, pySBeLT first constructs a bed of fixed particles of **'set_diam'** in both the downstream and cross-stream dimensions (one particle wide in the present build), and over a downstream domain length **'bed_length'**. Bed surface particles of **'particle_diam'** are then randomly placed at vertices between fixed bed particles until the**'particle_pack_frac'** is met. Vertices are defined by a contact point between two adjacent particles. The bed of surface particles is then separated into **'num_subregions'** set by the user. Subregion boundaries occur at domain locations set by a distance = **'bed_length'** / **'num_subregions'**. Following construction of the bed surface the forward simulations are ready to commence.

Simulation iterations involve three steps (Fig. 1): (1) the number of particle entrainment events per **'num_subregions'** are drawn from a Poisson pmf, and this is done randomly for each numerical step up to **'iterations'**; (2) surface particles from each subregion are randomly selected for entrainment, and if there are insufficient surface particles available for entrainment, then all available particles are moved; (3) each entrained particle moves a distance according to a randomly sampled value from either the normal or lognormal distribution (see THEORY.md for more details), and is placed at the nearest vertex between two particles that is available for placement. Placed particles are permitted to stack up to the **'level_limit'** in height. Particles are not permitted to travel to the same available vertex. To stop this from occuring the entrained particles are moved in random order and once a particle is placed on a vertex, that vertex is no longer considered available for the present iteration. Travel distances of particles that exceed **'bed_length'** are returned and queued at the upstream boundary, and are introduced back into the domain at the next numerical step according to travel distance sampling described above. This specifially means that the particle travel distance which resulted in crossing of the downstream domain does not influence the travel distance of the particle when queued at the upstream domain–a new travel distance for such particles will be sampled during the next numerical step. This overall process repeats for the specified **'iterations'**.

pySBeLT tracks a number of different parameters through a simulation: the vertical and horizontal positions of every particle center, the randomly sampled number of entrainment events, the number of particles actually entrained, the actual particle travel distance, the particle 'age', or the number of numerical steps since last entrainment for every particle, and the number of particles which cross all boundaries, i.e. sub-region and downstream at x_max. All values, or the values needed to derive this information, are stored in HDF5 data files using the h5py package (Collette, 2013).

pySBeLT produces a time varying signal of particle flux counted at the downstream domain (as well as internal subregion domains), with a particle bed that changes through particle stacking and pile removal, and downstream motions of travel distance (Fig. 2). An implication of particle stacking within the context of the pySBeLT stochastic framework is a time varying signal of the average "particle age", as well as the average "particle age range", defined as the difference of the maximum and minimum particle ages. The model can be readily modified to simulate kinematics using different probability distributions (see THEORY.md for more details), or examining particle age dynamics for deeper beds of particles available for transport. The relatively simple parameterization of pySBeLT execution also makes it suitable for use as a teaching tool within advanced undergraduate and graduate courses emphasizing bed load sediment transport.
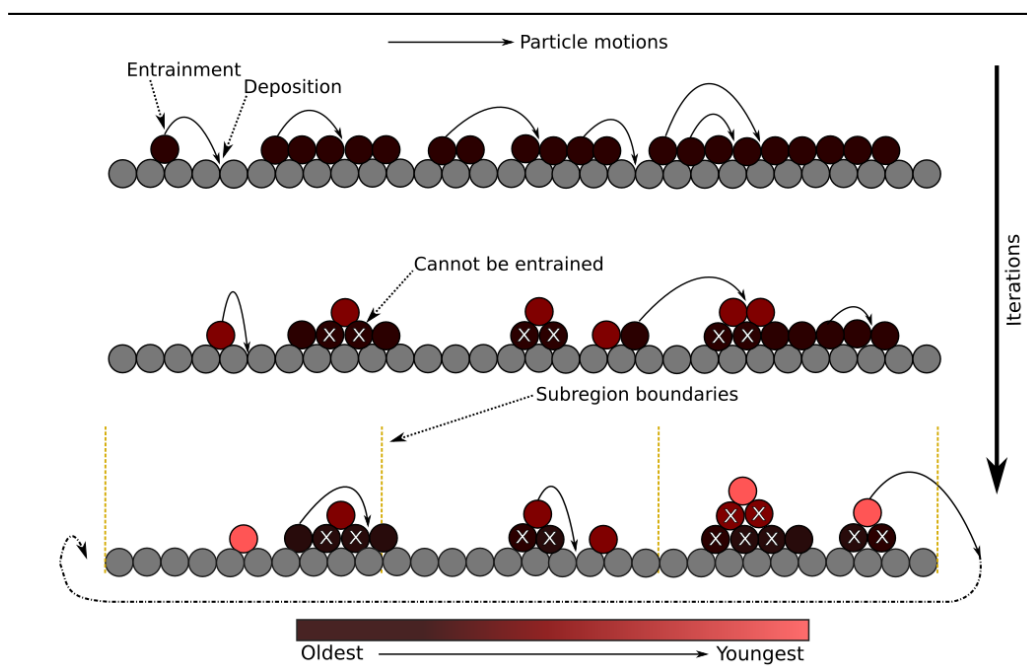
Figures

Figure 1. Graphic illustrating the three steps of particle transport modelling by py_SBeLT. The **'level_limit'** in height is set to 3 in the graphic.
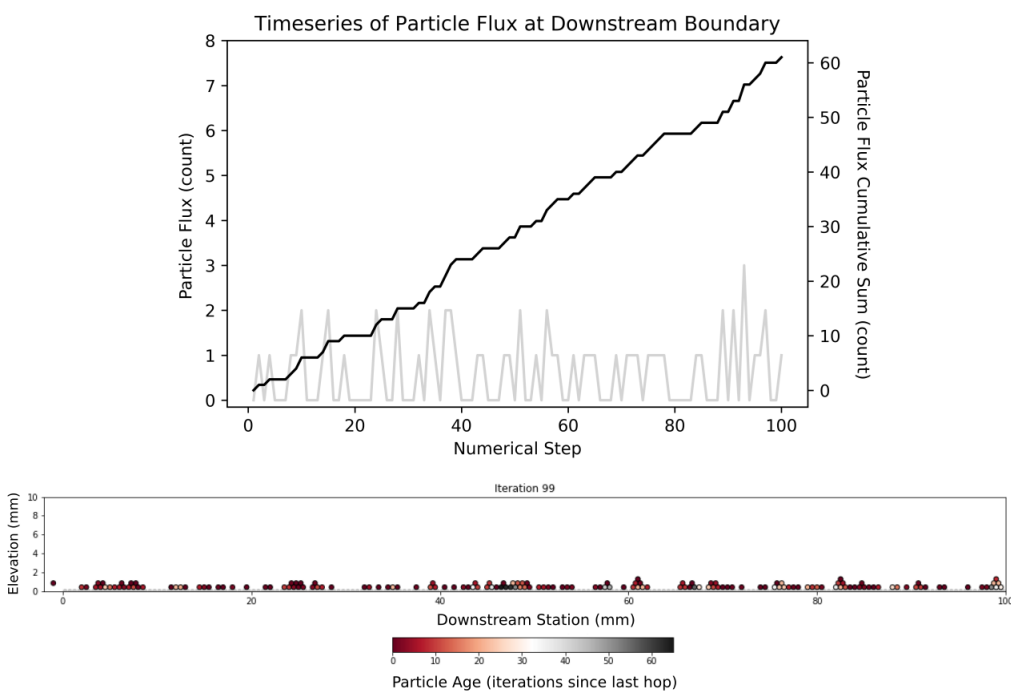

Figure 2. Example py_SBeLT output of particle flux at downstream boundary and particle bed configuration at numerical step 100

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