The Journal of Open Source Software

PropPy – Correlated random walk propagation of cosmic rays in magnetic turbulence

P. Reichherzer [*1,2,3] **and J. Becker Tjus** [1,2]

1 Ruhr-Universität Bochum, D-44801 Bochum, Germany **2** Ruhr Astroparticle and Plasma Physics Center, D-44780 Bochum, Germany **3** Université Paris-Saclay, F-91190 Gif-sur-Yvette, France

DOI: 10.21105/joss.04243

Software

- Review
- Repository
- Archive

Editor: Christina Hedges

Reviewers:

- @carmeloevoli
- @amitseta90

Submitted: 09 February 2022
Published: 02 June 2022

Summary

PropPy is an open-source Python software package for propagating charged high-energy particles (cosmic rays, CRs) in a turbulent magnetic field. Its modular architecture comprises various modules for sources, magnetic fields, propagators, and observers covering a wide range of applications.

When compared to codes that solve the equation of motion (EOM) in each propagation step, our propagation is based on a correlated random walk (CRW) in Cartesian (for isotropic diffusion) or cylindrical (for anisotropic diffusion) coordinates, which makes each simulation step significantly faster. This novel approach is justified by the fact that a transport equation can be derived via the formulation of the CRW (see theory section below), which is used in analytical descriptions of particle transport (Effenberger & Litvinenko, 2014; Y. E. Litvinenko et al., 2015; Yuri E. Litvinenko & Noble, 2013; Tautz & Lerche, 2016):

$$\frac{\partial f}{\partial t} + \sum_i \tau_i \frac{\partial^2 f}{\partial t^2} = \sum_i \kappa_i \frac{\partial^2 f}{\partial x_i^2}, \tag{1}$$

where i indicates the three spatial directions, τ_i denotes the time scale for particles to become diffusive, and κ_i is the diffusion coefficient, from which the relevant parameters of the CRW can be determined.

Besides the analytical verification of the CRW ansatz, comparison simulations between PropPy and an established cosmic-ray propagation software, CRPropa, are presented. These tests show that both approaches are comparable in terms of the statistical properties such as the running diffusion coefficient and the escape times from regions such as are relevant and present in many astrophysical environments.

This makes PropPy a high-performance software package for the simulation of charged particles in turbulent magnetic fields. This is especially true for compact objects and transient events with short time scales, such as gamma-ray bursts (GRBs), active galactic nuclei (AGN) flares, where the accurate description of the initial particle propagation is crucial. Fast simulations of transient events can help analyze observations and provide information to evaluate the need for follow-up observations in the context of real-time multimessenger astrophysics (Reichherzer et al., 2021).

Statement of need

Understanding the transport of charged high-energy particles in turbulent magnetic fields is essential for resolving the long-standing question of their extragalactic origin. The transport properties of cosmic rays are relevant in many ways:

*first author

- In cosmic-ray sources, the transport properties determine their residence time in the sources and thus the interaction processes leading to the production of secondary particles (Becker Tjus & Merten, 2020).
- Due to the enormous distance from sources to our galaxy, cosmic rays have to travel through the turbulent intergalactic medium (Alves Batista et al., 2018; Schlegel et al., 2020).
- In our galaxy, the galactic magnetic field influences their trajectory (Reichherzer, Merten, et al., 2022) and, finally, their arrival in the Earth's atmosphere due to anisotropic diffusion (Effenberger et al., 2012).

Analytical theories have been developed over the last century (Jokipii, 1966; Schlickeiser, 2015; Shalchi, 2021; Zweibel, 2013) to describe the transport of cosmic rays. However, these theories are often limited by strongly simplifying assumptions concerning the transport of charged particles in turbulent magnetic fields. To overcome these limitations, propagation codes with dedicated cosmic-ray-transport simulations have been developed over the last decades (Casse et al., 2001; Giacalone & Jokipii, 1999; Reichherzer, Tjus, et al., 2022; Reichherzer et al., 2020; Shukurov et al., 2017). In EOM propagation methods, particles are moved stepwise, with the next step always determined based on the solution of the EOM with the external force as the Lorentz force only taking into account magnetic fields. Note the magnetic field must be computed for each propagation step for all particle positions, a process that is typically time-consuming in numerical simulations. This is especially relevant when the particles are highly diffusive, i.e.,when the size of the propagation environment L exceeds the gyro radius of the particle $r_g \ll L$. A much more efficient method, the diffusive approach, is based on the statistical properties of the particles and exploits their theoretical description via a transport equation (Merten et al., 2017). In the limit of infinitely large times, diffusive transport occurs for all charged particles in isotropic turbulence. In the transport equation, the diffusion tensor implicitly contains all statistic properties. A major drawback of this approach is that can only model the transport of charged particles over large time scales so that the particles have enough time to become diffusive (Becker Tjus et al., 2022).

To tackle this issue and meet the need for realistic and fast simulations of the sources of cosmic rays, we present the PropPy software. Our software applies the approach of the CRW, where statistical aspects are used for speed-up while also providing a good description of the initial phase. Additionally, the properties of the CRW can be determined directly from the diffusion tensor and the gyration radius of the particle.

Comparison

Simulations are used for describing as accurately as possible the particle transport that has an impact on numerous observable multimessenger signatures. In the following comparison, we focus on the transport properties in these sources, which are described by the diffusion coefficient.

In principle, the CRW propagation method implemented in PropPy can be applied wherever other propagation codes for charged particles such as CRPropa (Alves Batista et al., 2022, 2016), DRAGON (Evoli et al., 2017), GALPROP (Strong & Moskalenko, 1998) are already in use. However, the advantages of PropPy are especially in the high performance and the accurate description of statistical transport properties also for the initial transport regime, which is not possible for pure diffusive propagation approaches.

Since CRPropa is the only code that supports both EOM and diffusive propagation methods with anisotropic diffusion coefficients, this software (version: CRPropa 3.1.7) is used for comparison simulations with PropPy.

We compare the performance of PropPy with the two different propagation methods implemented in CRPropa, which are:

1. Solving the EOM, using either the Boris-Push (BP) or the Cash-Karp (CK) algorithm.
2. Solving Stochastic Differential Equations (SDE). For this method, no turbulence has to be generated, but only the diffusion coefficient has to be inputted, which already contains the information on how the particles move statistically in the turbulence.

The comparison of the three different approaches diffusive, EOM, and CRW shows that CRW simulation results are in good agreement with EOM simulations, while being considerably faster.

Acknowledgements

We acknowledge support from funding from the German Science Foundation DFG, within the Collaborative Research Center SFB1491 "Cosmic Interacting Matters - From Source to Signal". PR acknowledge support by Institut Pascal at Université Paris-Saclay during the Paris-Saclay Astroparticle Symposium 2021, with the support of the P2IO Laboratory of Excellence (programme "Investissements d'avenir" ANR-11-IDEX-0003-01 Paris-Saclay and ANR-10-LABX-0038), the P2I research departments of the Paris-Saclay university, as well as IJCLab, CEA, IPhT, APPEC, the IN2P3 master projet UCMN and EuCAPT. Special thanks to L. Schlegel, F. Schüssler, J. Suc, and E.G. Zweibel for valuable discussions. We appreciate the efforts of editor C. Hedges and the reviewers C. Evoli and A. Seta for reviewing the software package and the paper.

References

Alves Batista, R., Becker Tjus, J., Dörner, J., Dundovic, A., Eichmann, B., Frie, A., Heiter, C., Hoerbe, M. R., Kampert, K. H., Merten, L., Müller, G., Reichherzer, P., Saveliev, A., Schlegel, L., Sigl, G., van Vliet, A., & Winchen, T. (2022). CRPropa 3.2: a framework for high-energy astroparticle propagation. *37th International Cosmic Ray Conference. 12-23 July 2021. Berlin*, 978. https://arxiv.org/abs/2107.01631

Alves Batista, R., de Gouveia Dal Pino, E. M., Dolag, K., & Hussain, S. (2018). Cosmic-ray propagation in the turbulent intergalactic medium. *arXiv e-Prints*, arXiv:1811.03062. https://arxiv.org/abs/1811.03062

Alves Batista, R., Dundovic, A., Erdmann, M., Kampert, K.-H., Kuempel, D., Müller, G., Sigl, G., Vliet, A. van, Walz, D., & Winchen, T. (2016). CRPropa 3—a public astrophysical simulation framework for propagating extraterrestrial ultra-high energy particles. *JCAP*, *2016*(5), 038. https://doi.org/10.1088/1475-7516/2016/05/038

Becker Tjus, J., Hörbe, M., Jaroschewski, I., Reichherzer, P., Rhode, W., Schroller, M., & Schüssler, F. (2022). Propagation of Cosmic Rays in Plasmoids of AGN Jets-Implications for Multimessenger Predictions. *Physics*, *4*(2), 473–490. https://doi.org/10.3390/physics4020032

Becker Tjus, J., & Merten, L. (2020). Closing in on the origin of Galactic cosmic rays using multimessenger information. *Physrep*, *872*, 1–98. https://doi.org/10.1016/j.physrep.2020.05.002

Casse, F., Lemoine, M., & Pelletier, G. (2001). Transport of cosmic rays in chaotic magnetic fields. *PRD*, *65*(2), 023002. https://doi.org/10.1103/PhysRevD.65.023002

Effenberger, F., Fichtner, H., Scherer, K., & Büsching, I. (2012). Anisotropic diffusion of Galactic cosmic ray protons and their steady-state azimuthal distribution. *AA*, *547*, A120. https://doi.org/10.1051/0004-6361/201220203

Effenberger, F., & Litvinenko, Y. (2014). The Diffusion Approximation versus the Telegraph Equation for Modeling Solar Energetic Particle Transport with Adiabatic Focusing. I.

Isotropic Pitch-angle Scattering. *ApJ*, *783*(1), 15. https://doi.org/10.1088/0004-637X/783/1/15

Evoli, C., Gaggero, D., Vittino, A., Di Bernardo, G., Di Mauro, M., Ligorini, A., Ullio, P., & Grasso, D. (2017). Cosmic-ray propagation with DRAGON2: I. numerical solver and astrophysical ingredients. *JCAP*, *2017*(2), 015. https://doi.org/10.1088/1475-7516/2017/02/015

Giacalone, J., & Jokipii, J. R. (1999). The Transport of Cosmic Rays across a Turbulent Magnetic Field. *ApJ*, *520*(1), 204–214. https://doi.org/10.1086/307452

Jokipii, J. R. (1966). Cosmic-Ray Propagation. I. Charged Particles in a Random Magnetic Field. *ApJ*, *146*, 480. https://doi.org/10.1086/148912

Litvinenko, Y. E., Effenberger, F., & Schlickeiser, R. (2015). The Telegraph Approximation for Focused Cosmic-Ray Transport in the Presence of Boundaries. *ApJ*, *806*(2), 217. https://doi.org/10.1088/0004-637X/806/2/217

Litvinenko, Yuri E., & Noble, P. L. (2013). A Numerical Study of Diffusive Cosmic-Ray Transport with Adiabatic Focusing. *ApJ*, *765*(1), 31. https://doi.org/10.1088/0004-637X/765/1/31

Merten, L., Becker Tjus, J., Fichtner, H., Eichmann, B., & Sigl, G. (2017). CRPropa 3.1—a low energy extension based on stochastic differential equations. *JCAP*, *2017*(6), 046. https://doi.org/10.1088/1475-7516/2017/06/046

Reichherzer, P., Becker Tjus, J., Zweibel, E. G., Merten, L., & Pueschel, M. J. (2020). Turbulence-level dependence of cosmic ray parallel diffusion. *MNRAS*, *498*(4), 5051–5064. https://doi.org/10.1093/mnras/staa2533

Reichherzer, P., Merten, L., Dörner, J., Becker Tjus, J., Pueschel, J., & Zweibel, E. G. (2022). Regimes of cosmic-ray diffusion in Galactic turbulence. *SN Applied Sciences*, *4*, 15. https://doi.org/10.1007/s42452-021-04891-z

Reichherzer, P., Schüssler, F., Lefranc, V., Yusafzai, A., Alkan, A. K., Ashkar, H., & Becker Tjus, J. (2021). Astro-COLIBRI-The COincidence LIBrary for Real-time Inquiry for Multi-messenger Astrophysics. *ApJs*, *256*(1), 5. https://doi.org/10.3847/1538-4365/ac1517

Reichherzer, P., Tjus, J. B., Zweibel, E. G., Merten, L., & Pueschel, M. J. (2022). Anisotropic cosmic ray diffusion in isotropic Kolmogorov turbulence. *MNRAS*. https://doi.org/10.1093/mnras/stac1408

Schlegel, L., Frie, A., Eichmann, B., Reichherzer, P., & Becker Tjus, J. (2020). Interpolation of Turbulent Magnetic Fields and Its Consequences on Cosmic Ray Propagation. *APJ*, *889*(2), 123. https://doi.org/10.3847/1538-4357/ab643b

Schlickeiser, R. (2015). Cosmic ray transport in astrophysical plasmas. *Physics of Plasmas*, *22*(9), 091502. https://doi.org/10.1063/1.4928940

Shalchi, A. (2021). Perpendicular Diffusion of Energetic Particles: A Complete Analytical Theory. *ApJ*, *923*(2), 209. https://doi.org/10.3847/1538-4357/ac2363

Shukurov, A., Snodin, A. P., Seta, A., Bushby, P. J., & Wood, T. S. (2017). Cosmic Rays in Intermittent Magnetic Fields. *ApJL*, *839*(1), L16. https://doi.org/10.3847/2041-8213/aa6aa6

Strong, A. W., & Moskalenko, I. V. (1998). Propagation of Cosmic-Ray Nucleons in the Galaxy. *ApJ*, *509*(1), 212–228. https://doi.org/10.1086/306470

Tautz, R. C., & Lerche, I. (2016). Application of the three-dimensional telegraph equation to cosmic-ray transport. *Research in Astronomy and Astrophysics*, *16*(10), 162. https://doi.org/10.1088/1674-4527/16/10/162

Zweibel, E. G. (2013). The microphysics and macrophysics of cosmic rays. *Physics of Plasmas*, *20*(5), 055501. https://doi.org/10.1063/1.4807033