

# iharm3D: Vectorized General Relativistic Magnetohydrodynamics

Ben S. Prather<sup>\*1, 2</sup>, George N. Wong<sup>1, 2</sup>, Vedant Dhruv<sup>1, 2</sup>, Benjamin R. Ryan<sup>3</sup>, Joshua C. Dolence<sup>3</sup>, Sean M. Ressler<sup>5</sup>, and Charles F. Gammie<sup>1, 2, 4</sup>

**1** Physics Department, University of Illinois at Urbana–Champaign, 1110 West Green Street, Urbana, IL 61801, USA **2** Illinois Center for Advanced Studies of the Universe **3** CCS-2, Los Alamos National Laboratory, P.O. Box 1663, Los Alamos, NM 87545, USA **4** Astronomy Department, University of Illinois at Urbana–Champaign, 1002 West Green Street, Urbana, IL 61801, USA **5** Kavli Institute for Theoretical Physics, University of California Santa Barbara, Kohn Hall, Santa Barbara, CA 93107, USA

DOI: [10.21105/joss.03336](https://doi.org/10.21105/joss.03336)

## Software

- [Review](#) ↗
- [Repository](#) ↗
- [Archive](#) ↗

**Editor:** [Eloisa Bentivegna](#) ↗

## Reviewers:

- [@bgiacomma](#)
- [@cpalenzuela](#)

**Submitted:** 14 May 2021

**Published:** 14 October 2021

## License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](https://creativecommons.org/licenses/by/4.0/)).

## iharm3D Functionality and Purpose

iharm3D<sup>1</sup> is an open-source C code for simulating black hole accretion systems in arbitrary stationary spacetimes using ideal general-relativistic magnetohydrodynamics (GRMHD). It is an implementation of the HARM (“High Accuracy Relativistic Magnetohydrodynamics”) algorithm outlined in [Gammie et al. \(2003\)](#) with updates as outlined in [McKinney & Gammie \(2004\)](#) and [Noble et al. \(2006\)](#). The code is most directly derived from [Ryan et al. \(2015\)](#) but with radiative transfer portions removed. HARM is a conservative finite-volume scheme for solving the equations of ideal GRMHD, a hyperbolic system of partial differential equations, on a logically Cartesian mesh in arbitrary coordinates.

## Statement of Need

Numerical simulations are crucial in modeling observations of active galactic nuclei, such as the recent horizon-scale results from the Event Horizon Telescope and GRAVITY collaborations. The computational simplicity of ideal GRMHD enables the generation of long, high-resolution simulations and broad parameter-exploration studies that can be compared to observations for parameter inference.

Multiple codes already exist for solving the ideal GRMHD equations on regular Eulerian meshes in 3D, including:

- Athena++ ([Stone et al. \(2020\)](#), [White et al. \(2016\)](#))
- BHAC ([Porth et al. \(2017\)](#))
- Cosmos++ ([Anninos et al. \(2005\)](#), [Fragile et al. \(2012\)](#), [Fragile et al. \(2014\)](#))
- ECHO ([Londrillo & Zanna \(2000\)](#), [Londrillo & Zanna \(2004\)](#))
- H-AMR ([Matthew Liska et al. \(2019\)](#), [M. Liska et al. \(2020\)](#))
- HARM-Noble ([Noble et al. \(2006\)](#), [Noble et al. \(2009\)](#), [Noble et al. \(2012\)](#), [Zilhão & Noble \(2014\)](#), [Bowen et al. \(2018\)](#))
- IllinoisGRMHD ([Etienne et al. \(2015\)](#))

\*Corresponding author

<sup>1</sup><https://github.com/AFD-Illinois/iharm3d>

- KORAL (Sądowski et al. (2013), Sądowski et al. (2014))
- GRHydro (Mösta et al. (2014))
- Spritz (Cipolletta et al. (2020), Cipolletta et al. (2021))

As the length of this list illustrates, the field of GRMHD simulation is now well established, and many codes now exist to serve different needs. These codes can be distinguished by the trade-offs they make in prioritizing speed, simplicity, and generality, with the latter encompassing, e.g., support for dynamical spacetimes, adaptive mesh refinement, or higher-order integration schemes.

In particular, `iharm3D` aims to be a simple and fast code capable of simulating the original systems of interest when designing HARM, even at the cost of features aimed at more general applicability. It provides a fast and scalable update to HARM, but maintains the conventions and structure of the original described in Gammie et al. (2003). The result is a code that is relatively easy to understand and modify, yet capable of running simulations at state-of-the-art scale.

## Implementation Notes

In MHD, uncorrected discretization errors inevitably lead to violations of the no-monopoles condition  $\nabla \cdot B = 0$ . As in the original HARM implementation, `iharm3D` uses the “Flux-CT” scheme for cell-centered constrained transport outlined in Tóth (2000).

`iharm3D` also retains numerical evaluation of all metric-dependent quantities, allowing trivial modification of the coordinate system or background spacetime so long as the line element is available in analytic form. This can be used as a form of static mesh refinement, since the coordinates can be adapted to place resolution in areas of interest (e.g., near the accretion disk midplane).

In GRMHD, “conserved” variables (energy and momentum densities) are complicated analytic functions of “primitive” variables (density, pressure, and velocity). Conserved variables are stepped forward in time and then inversion to primitives is done numerically. `iharm3D` uses the “ $1D_W$ ” scheme outlined in Noble et al. (2006).

As the equations of ideal GRMHD are rescalable, any consistent set of units may be chosen to evolve them in the code. For numerical stability, when simulating accretion systems we choose units in which  $GM = c = 1$  with  $M$  the mass of the central object, and scale the density of the initial conditions such that the maximum value of  $\rho$  is 1.

To model a collisionless plasma, `iharm3D` implements an optional means of tracking and partitioning dissipation into ions and electrons (Ressler et al. (2015)). Currently the code implements five different heating models, described in Howes (2010), Kawazura et al. (2019), Werner et al. (2018), Rowan et al. (2017), and Sharma et al. (2007).

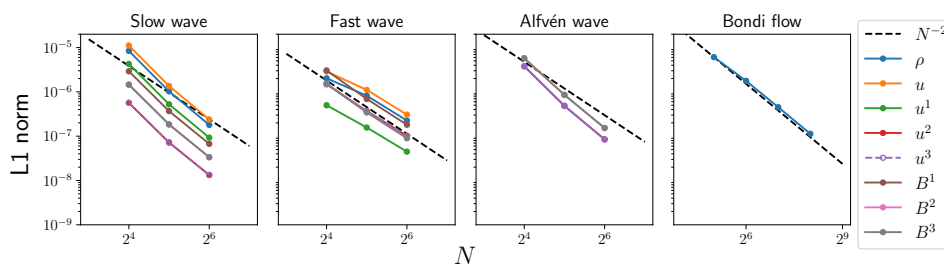
To avoid catastrophic failures caused by discretization error, especially in low density regions, fluid variables are bounded at the end of each step. Typically, the bounds in black hole accretion problems are enforced as follows:

- Density  $\rho > 10^{-6}k$ , for  $k \equiv \frac{1}{r^2(1+r/10)}$ , with  $r$  the radial coordinate,
- Internal energy density  $u > 10^{-8}k^\gamma$  where  $\gamma \equiv$  adiabatic index,
- $\rho$  and  $u$  are incremented until  $\sigma \equiv \frac{2P_b}{\rho} < 400$  and  $\beta \equiv \frac{P_{gas}}{P_b} > 2.5 \times 10^{-5}$  where  $P_b \equiv \frac{b^2}{2}$  is the magnetic pressure,
- $\rho$  is incremented until  $\frac{u}{\rho} < 100$ ,
- When evolving electron temperatures,  $u$  is decremented until  $\frac{P_{gas}}{\rho^\gamma} < 3$ ,
- Velocity components are downscaled until Lorentz factor  $\Gamma \equiv \frac{1}{\sqrt{1-v^2}} < 50$ .

Global disk simulations inevitably invoke these bounds, most frequently those on  $\sigma$  and  $\Gamma$ .

## Tests

The convergence properties of HARM are well-studied in Gammie et al. (2003). `iharm3D` implements most of the tests presented in that paper as integration and regression tests. Figure 1 shows convergence results for linear modes and for un-magnetized Bondi flow.

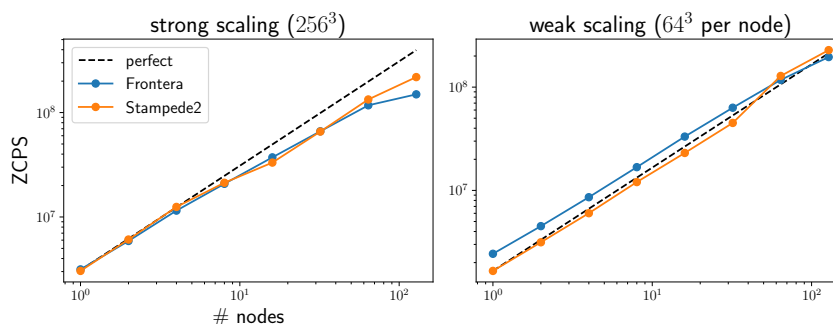


**Figure 1:** Results of convergence tests with `iharm3D`'s main branch, plotting L1 norm of the difference between the computed solution and the analytic or stable result with increasing domain size. Wave solutions were performed on a 3D cubic grid  $N$  zones to one side, the Bondi accretion problem was performed on a logically Cartesian 2D square grid  $N$  zones on one side.

`iharm3D` implements three additional tests which check that fluid evolution is identical for different domain decompositions: one which initializes a new fluid state, one which restarts from a checkpoint file, and one comparing the initialized state to an equivalent checkpoint file.

## Scaling

Key `iharm3D` routines are written for effective compiler vectorization and prioritize simple memory access patterns to make good use of high memory bandwidth. Originally developed for Intel Knights Landing (KNL) chips on the Stampede2 supercomputer at Texas Advanced Computing Center (TACC), `iharm3D` also runs efficiently on TACC Frontera, which uses traditional Cascade Lake (CLX) CPUs. Figure 2 presents scaling results for `iharm3D` on both Stampede2 and Frontera.



**Figure 2:** Strong and weak scaling performance of `iharm3D`. Performance is measured in zones advanced by one cycle each second (Zone-Cycles per Second). In the strong scaling test, an accretion torus problem of constant total size  $256^3$  was split among all nodes; in the weak scaling test, the total problem size was varied to ensure that a mesh block of size  $64^3$  was allocated to each node.

## Research projects using `iharm3D`

`iharm3D` is one of several GRMHD codes used by the EHT Collaboration to produce its library of fluid simulations. Images produced from this library were used for validation tests in [Event Horizon Telescope Collaboration et. al. \(2019a\)](#) and [Event Horizon Telescope Collaboration et. al. \(2021a\)](#) and for interpretation of the M87 EHT results in total intensity ([Event Horizon Telescope Collaboration et. al. \(2019b\)](#), [Event Horizon Telescope Collaboration et. al. \(2019c\)](#)) and polarization ([Event Horizon Telescope Collaboration et. al. \(2021b\)](#)).

Papers making use of the results of `iharm3D` simulations include [Porth et al. \(2019\)](#), [Johnson et al. \(2020\)](#), [Gold et al. \(2020\)](#), [Palumbo et al. \(2020\)](#), [Lin et al. \(2020\)](#), [Ricarte et al. \(2020\)](#), [Wielgus et al. \(2020\)](#), [Tiede et al. \(2020\)](#), and [Gelles et al. \(2021\)](#).

## Acknowledgements

This work was supported by National Science Foundation grants AST 17-16327, OISE 17-43747, AST 20-07936, AST 20-34306, and PHY 17-48958, by a Donald C. and F. Shirley Jones Fellowship to G.N.W., by the Gordon and Betty Moore Foundation through Grant GBMF7392, and by the US Department of Energy through Los Alamos National Laboratory. Los Alamos National Laboratory is operated by Triad National Security, LLC, for the National Nuclear Security Administration of the US Department of Energy (Contract No. 89233218CNA000001). This work has been assigned a document release number LA-UR-21-23714.

This work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by National Science Foundation grant number ACI-1548562, specifically the XSEDE resource Stampede2 at the Texas Advanced Computing Center (TACC) through allocation TG-AST170024. The authors acknowledge the Texas Advanced Computing Center at The University of Texas at Austin for providing HPC resources that have contributed to the research results reported within this paper.

## References

- Anninos, P., Fragile, P. C., & Salmonson, J. D. (2005). Cosmos++: Relativistic Magnetohydrodynamics on Unstructured Grids with Local Adaptive Refinement. *The Astrophysical Journal*, 635(1), 723. <https://doi.org/10.1086/497294>
- Bowen, D. B., Mewes, V., Campanelli, M., Noble, S. C., Krolik, J. H., & Zilhão, M. (2018). Quasi-periodic Behavior of Mini-disks in Binary Black Holes Approaching Merger. *The Astrophysical Journal Letters*, 853(1), L17. <https://doi.org/10.3847/2041-8213/aaa756>
- Cipolletta, F., Kalinani, J. V., Giacomazzo, B., & Ciolfi, R. (2020). Spritz: A new fully general-relativistic magnetohydrodynamic code. *Classical and Quantum Gravity*, 37, 135010. <https://doi.org/10.1088/1361-6382/ab8be8>
- Cipolletta, F., Kalinani, J. V., Giangrandi, E., Giacomazzo, B., Ciolfi, R., Sala, L., & Giudici, B. (2021). Spritz: General relativistic magnetohydrodynamics with neutrinos. *Classical and Quantum Gravity*, 38, 085021. <https://doi.org/10.1088/1361-6382/abebb7>
- Etienne, Z. B., Paschalidis, V., Haas, R., Mösta, P., & Shapiro, S. L. (2015). IllinoisGRMHD: An open-source, user-friendly GRMHD code for dynamical spacetimes. *Classical and Quantum Gravity*, 32(17), 175009. <https://doi.org/10.1088/0264-9381/32/17/175009>
- Event Horizon Telescope Collaboration et. al. (2021a). First M87 Event Horizon Telescope Results. VII. Polarization of the Ring. *910*(1), L12. <https://doi.org/10.3847/2041-8213/abe71d>

- Event Horizon Telescope Collaboration et. al. (2021b). First M87 Event Horizon Telescope Results. VIII. Magnetic Field Structure near The Event Horizon. *910*(1), L13. <https://doi.org/10.3847/2041-8213/abe4de>
- Event Horizon Telescope Collaboration et. al. (2019a). First M87 Event Horizon Telescope Results. IV. Imaging the Central Supermassive Black Hole. *875*(1), L4. <https://doi.org/10.3847/2041-8213/ab0e85>
- Event Horizon Telescope Collaboration et. al. (2019b). First M87 Event Horizon Telescope Results. V. Physical Origin of the Asymmetric Ring. *875*(1), L5. <https://doi.org/10.3847/2041-8213/ab0f43>
- Event Horizon Telescope Collaboration et. al. (2019c). First M87 Event Horizon Telescope Results. VI. The Shadow and Mass of the Central Black Hole. *875*(1), L6. <https://doi.org/10.3847/2041-8213/ab1141>
- Fragile, P. C., Gillespie, A., Monahan, T., Rodriguez, M., & Anninos, P. (2012). Numerical Simulations of Optically Thick Accretion onto a Black Hole. I. Spherical Case. *The Astrophysical Journal Supplement Series*, *201*, 9. <https://doi.org/10.1088/0067-0049/201/2/9>
- Fragile, P. C., Olejar, A., & Anninos, P. (2014). Numerical Simulations of Optically Thick Accretion onto a Black Hole. II. Rotating Flow. *The Astrophysical Journal*, *796*, 22. <https://doi.org/10.1088/0004-637X/796/1/22>
- Gammie, C. F., McKinney, J. C., & Tóth, G. (2003). HARM: A Numerical Scheme for General Relativistic Magnetohydrodynamics. *The Astrophysical Journal*, *589*(1), 444. <https://doi.org/10.1086/374594>
- Gelles, Z., Prather, B. S., Palumbo, D. C. M., Johnson, M. D., Wong, G. N., & Georgiev, B. (2021). The Role of Adaptive Ray Tracing in Analyzing Black Hole Structure. *The Astrophysical Journal*, *912*(1), 39. <https://doi.org/10.3847/1538-4357/abee13>
- Gold, R., Broderick, A. E., Younsi, Z., Fromm, C. M., Gammie, C. F., Mościbrodzka, M., Pu, H.-Y., Bronzwaer, T., Davelaar, J., Dexter, J., Ball, D., Chan, C., Kawashima, T., Mizuno, Y., Ripperda, B., Akiyama, K., Alberdi, A., Alef, W., Asada, K., ... Event Horizon Telescope Collaboration. (2020). Verification of Radiative Transfer Schemes for the EHT. *897*(2), 148. <https://doi.org/10.3847/1538-4357/ab96c6>
- Howes, G. G. (2010). A prescription for the turbulent heating of astrophysical plasmas. *Monthly Notices of the Royal Astronomical Society*, *409*, L104–L108. <https://doi.org/10.1111/j.1745-3933.2010.00958.x>
- Johnson, M. D., Lupsasca, A., Strominger, A., Wong, G. N., Hadar, S., Kapec, D., Narayan, R., Chael, A., Gammie, C. F., Galison, P., Palumbo, D. C. M., Doeleman, S. S., Blackburn, L., Wielgus, M., Pesce, D. W., Farah, J. R., & Moran, J. M. (2020). Universal interferometric signatures of a black hole's photon ring. *Science Advances*, *6*(12), eaaz1310. <https://doi.org/10.1126/sciadv.aaz1310>
- Kawazura, Y., Barnes, M., & Schekochihin, A. A. (2019). Thermal disequilibrium of ions and electrons by collisionless plasma turbulence. *Proceedings of the National Academy of Science*, *116*, 771–776. <https://doi.org/10.1073/pnas.1812491116>
- Lin, J. Y.-Y., Wong, G. N., Prather, B. S., & Gammie, C. F. (2020). Feature Extraction on Synthetic Black Hole Images. *arXiv:2007.00794 [astro-Ph]*. <http://arxiv.org/abs/2007.00794>
- Liska, Matthew, Chatterjee, K., Tchekhovskoy, A., Yoon, D., Eijnatten, D. van, Hesp, C., Markoff, S., Ingram, A., & Klis, M. van der. (2019). H-AMR: A New GPU-accelerated GRMHD Code for Exascale Computing With 3D Adaptive Mesh Refinement and Local Adaptive Time-stepping. *arXiv:1912.10192 [astro-Ph]*. <http://arxiv.org/abs/1912.10192>

- Liska, M., Tchekhovskoy, A., & Quataert, E. (2020). Large-scale poloidal magnetic field dynamo leads to powerful jets in GRMHD simulations of black hole accretion with toroidal field. *Monthly Notices of the Royal Astronomical Society*, 494, 3656–3662. <https://doi.org/10.1093/mnras/staa955>
- Londrillo, P., & Zanna, L. D. (2000). High-Order Upwind Schemes for Multidimensional Magnetohydrodynamics. *The Astrophysical Journal*, 530(1), 508. <https://doi.org/10.1086/308344>
- Londrillo, P., & Zanna, L. del. (2004). On the divergence-free condition in Godunov-type schemes for ideal magnetohydrodynamics: The upwind constrained transport method. *Journal of Computational Physics*, 195, 17–48. <https://doi.org/10.1016/j.jcp.2003.09.016>
- McKinney, J. C., & Gammie, C. F. (2004). A Measurement of the Electromagnetic Luminosity of a Kerr Black Hole. *The Astrophysical Journal*, 611(2), 977. <https://doi.org/10.1086/422244>
- Mösta, P., Mundim, B. C., Faber, J. A., Haas, R., Noble, S. C., Bode, T., Löffler, F., Ott, C. D., Reisswig, C., & Schnetter, E. (2014). GRHydro: A new open-source general-relativistic magnetohydrodynamics code for the Einstein toolkit. *Classical and Quantum Gravity*, 31, 015005. <https://doi.org/10.1088/0264-9381/31/1/015005>
- Noble, S. C., Gammie, C. F., McKinney, J. C., & Del Zanna, L. (2006). Primitive Variable Solvers for Conservative General Relativistic Magnetohydrodynamics. *The Astrophysical Journal*, 641, 626–637. <https://doi.org/10.1086/500349>
- Noble, S. C., Krolik, J. H., & Hawley, J. F. (2009). DIRECT CALCULATION OF THE RADIATIVE EFFICIENCY OF AN ACCRETION DISK AROUND A BLACK HOLE. *The Astrophysical Journal*, 692(1), 411–421. <https://doi.org/10.1088/0004-637X/692/1/411>
- Noble, S. C., Mundim, B. C., Nakano, H., Krolik, J. H., Campanelli, M., Zlochower, Y., & Yunes, N. (2012). Circumbinary Magnetohydrodynamic Accretion into Inspiring Binary Black Holes. *The Astrophysical Journal*, 755, 51. <https://doi.org/10.1088/0004-637X/755/1/51>
- Palumbo, D. C. M., Wong, G. N., & Prather, B. S. (2020). Discriminating Accretion States via Rotational Symmetry in Simulated Polarimetric Images of M87. *The Astrophysical Journal*, 894(2), 156. <https://doi.org/10.3847/1538-4357/ab86ac>
- Porth, O., Chatterjee, K., Narayan, R., Gammie, C. F., Mizuno, Y., Anninos, P., Baker, J. G., Bugli, M., Chan, C., Davelaar, J., Del Zanna, L., Etienne, Z. B., Fragile, P. C., Kelly, B. J., Liska, M., Markoff, S., McKinney, J. C., Mishra, B., Noble, S. C., ... Collaboration, T. E. H. T. (2019). The Event Horizon General Relativistic Magnetohydrodynamic Code Comparison Project. *arXiv:1904.04923 [astro-Ph, Physics:gr-Qc]*. <https://doi.org/10.3847/1538-4365/ab29fd>
- Porth, O., Olivares, H., Mizuno, Y., Younsi, Z., Rezzolla, L., Moscibrodzka, M., Falcke, H., & Kramer, M. (2017). The Black Hole Accretion Code. *Computational Astrophysics and Cosmology*, 4(1). <https://doi.org/10.1186/s40668-017-0020-2>
- Ressler, S. M., Tchekhovskoy, A., Quataert, E., Chandra, M., & Gammie, C. F. (2015). Electron thermodynamics in GRMHD simulations of low-luminosity black hole accretion. *Monthly Notices of the Royal Astronomical Society*, 454, 1848–1870. <https://doi.org/10.1093/mnras/stv2084>
- Ricarte, A., Prather, B. S., Wong, G. N., Narayan, R., Gammie, C., & Johnson, M. D. (2020). Decomposing the internal faraday rotation of black hole accretion flows. *Monthly Notices of the Royal Astronomical Society*, 498, 5468–5488. <https://doi.org/10.1093/mnras/staa2692>

- Rowan, M. E., Sironi, L., & Narayan, R. (2017). Electron and Proton Heating in Transrelativistic Magnetic Reconnection. *The Astrophysical Journal*, 850, 29. <https://doi.org/10.3847/1538-4357/aa9380>
- Ryan, B. R., Dolence, J. C., & Gammie, C. F. (2015). Bhlight: General Relativistic Radiation Magnetohydrodynamics with Monte Carlo Transport. *The Astrophysical Journal*, 807(1), 31. <https://doi.org/10.1088/0004-637X/807/1/31>
- Sądowski, A., Narayan, R., McKinney, J. C., & Tchekhovskoy, A. (2014). Numerical simulations of super-critical black hole accretion flows in general relativity. *Monthly Notices of the Royal Astronomical Society*, 439(1), 503. <https://doi.org/10.1093/mnras/stt2479>
- Sądowski, A., Narayan, R., Tchekhovskoy, A., & Zhu, Y. (2013). Semi-implicit scheme for treating radiation under M1 closure in general relativistic conservative fluid dynamics codes. *Monthly Notices of the Royal Astronomical Society*, 429(4), 3533. <https://doi.org/10.1093/mnras/sts632>
- Sharma, P., Quataert, E., Hammett, G. W., & Stone, J. M. (2007). Electron Heating in Hot Accretion Flows. *The Astrophysical Journal*, 667, 714–723. <https://doi.org/10.1086/520800>
- Stone, J. M., Tomida, K., White, C. J., & Felker, K. G. (2020). The Athena++ Adaptive Mesh Refinement Framework: Design and Magnetohydrodynamic Solvers. *The Astrophysical Journal Supplement Series*, 249, 4. <https://doi.org/10.3847/1538-4365/ab929b>
- Tiede, P., Broderick, A. E., & Palumbo, D. C. M. (2020). Variational Image Feature Extraction for the EHT. *arXiv:2012.07889 [astro-Ph]*. <http://arxiv.org/abs/2012.07889>
- Tóth, G. (2000). The  $\nabla \cdot \mathbf{B} = 0$  Constraint in Shock-Capturing Magnetohydrodynamics Codes. *Journal of Computational Physics*, 161(2), 605–652. <https://doi.org/10.1006/jcph.2000.6519>
- Werner, G. R., Uzdensky, D. A., Begelman, M. C., Cerutti, B., & Nalewajko, K. (2018). Non-thermal particle acceleration in collisionless relativistic electron-proton reconnection. *Monthly Notices of the Royal Astronomical Society*, 473, 4840–4861. <https://doi.org/10.1093/mnras/stx2530>
- White, C. J., Stone, J. M., & Gammie, C. F. (2016). An Extension of the Athena++ Code Framework for GRMHD Based on Advanced Riemann Solvers and Staggered-mesh Constrained Transport. *The Astrophysical Journal Supplement Series*, 225(2), 22. <https://doi.org/10.3847/0067-0049/225/2/22>
- Wielgus, M., Akiyama, K., Blackburn, L., Chan, C., Dexter, J., Doeleman, S. S., Fish, V. L., Issaoun, S., Johnson, M. D., Krichbaum, T. P., Lu, R.-S., Pesce, D. W., Wong, G. N., Bower, G. C., Broderick, A. E., Chael, A., Chatterjee, K., Gammie, C. F., Georgiev, B., ... Zhu, Z. (2020). Monitoring the Morphology of M87\* in 2009–2017 with the Event Horizon Telescope. *901*(1), 67. <https://doi.org/10.3847/1538-4357/abac0d>
- Zilhão, M., & Noble, S. C. (2014). Dynamic fisheye grids for binary black hole simulations. *Classical and Quantum Gravity*, 31(6), 065013. <https://doi.org/10.1088/0264-9381/31/6/065013>