SAILS: Spectral Analysis In Linear Systems

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Summary & Background

Autoregressive modelling provides a powerful and flexible parametric approach to modelling uni- or multi-variate time-series data. AR models have mathematical links to linear time-invariant systems, digital filters and Fourier based frequency analyses. As such, a wide range of time-domain and frequency-domain metrics can be readily derived from the fitted autoregressive parameters. These approaches are fundamental in a wide range of science and engineering fields and still undergoing active development. SAILS (Spectral Analysis in Linear Systems) is a python package which implements such methods and provides a basis for both the straight-forward fitting of AR models as well as exploration and development of newer methods, such as the decomposition of autoregressive parameters into eigenmodes.

Package Features

The SAILS toolbox is designed to work with time-series data from any form of application, but was originally written by the authors with an intended use in human neuroscience. The package provides functionality for model fitting, (for example Ordinary Least-Squares and Vieira-Morf (Lawrence Marple Jr, 1987) approaches), model selection (Akaike's Information Criterion and Bayesian Information Criterion), model validation (e.g. Stability Index (Lütke-pohl, 2007), Durbin-Watson criteria (Durbin & Watson, 1950) and Percent Consistency (Ding et al., 2000)) Once fitted, a range of spectral features such as the power spectral density and the transfer function can be estimated from the fitted model parameters. An additional range of connectivity metrics can be computed from multivariate models - including Magnitude Squared Coherence, Partial Coherence, Directed Transfer Function (Kaminski & Blinowska, 1991), Partial Directed Coherence (Baccalá & Sameshima, 2001) and Isolated Effective Coherence (Pascual-Marqui et al., 2014). SAILS is written in a modular form and designed to be easily extensible. The authors intend to continue expanding the range of model fitting and connectivity methods as they are developed.

Advanced exploration of the spectral content of the model is provided via a modal decomposition of the fitted autoregressive parameters (Neumaier & Schneider, 2001), an alternative to analyses which require the use of Fourier transform. This method can provide an intuitive summary of the frequency content of the system via a set of oscillatory modes - each defined by a peak resonant frequency, a damping time and a projection into the full network. The transfer function of the system can be computed by linear summation of these modal parameters.

The software also includes plotting routines to make examining data easier for the user. These range from methods to assist in plotting matrices of frequency information across multiple nodes to routines to simplify the use of external plotting programs such as Circos (Krzywinski M, 2009) for circular plots. Examples of such plots are included in the Screenshots section of this paper.

Target Audience

SAILS is aimed for use in scientific and engineering data analysis of multivariate oscillating systems. As this opens up a wide range of potential use cases, we include a incomplete set of examples for illustration. An economist might be interested to use power spectra to quantify oscillatory or cyclical changes in economic variables such as financial markets. A biomedical engineer analysing M/EEG data may be interested in the spectral connectivity metrics for quantifying how oscillatory synchrony in brain networks changes between clinical populations. Similarly, a cognitive neuroscientist could use the sliding window model fits on MEG data to explore how brain networks change during a cognitive task. As a final example, an engineer or physicist may use the functions to explore resonant vibrations in a physical system. In practice, these features can be used in on any time-series to quantify oscillatory features, their spatial distribution and network interactions.

State of the field

Several Python packages provide related functionality to SAILS. Firstly, there are a range of packages implementing multivariate regression fits (including numpy.linalg and statsmodels.tsa). SAILS is readily extensible to work with model fits from other packages by creating a class inheriting from AbstractLinearModel and implementing the fit method to call the external package. Some spectral connectivity measures (based on multi-taper or Wavelet transforms and not currently including MVAR based measures such as Partial Directed Coherence) are implemented in MNE-python, a specialised electrophysiology analysis package. Other packages (such as pyspectrum) provide autoregressive spectrum estimation methods but are not readily extensible to multivariate datasets or connectivity metrics. Finally, some packages implement varying range of autoregressive connectivity estimates (such as nitime, SCoT, connectivipy and spectral_connectivity). SAILS provides overlapping functionality to these packages alongside advanced methods such as the modal decomposition.

Availability and Installation

SAILS is released under a GPLv2 or later license.

The toolbox is available on PyPi and can therefore be installed using pip using `pip install sails` or similar. It has minimal dependencies although if users wish to produce circular plots, circos will need to be installed either from the website or via the users package manager on systems such as Debian or Ubuntu.

The package is designed for use under Python 3. Most functions will work under Python 2, but the authors no longer actively test under this version.

Implementation and Usage

Tutorials and documentation regarding the use of the module are available at https://sails.readthedocs.org. Development and bug tracking is hosted at https://vcs.ynic.york.ac.uk/analysis/sails.

The majority of workflows can be summarised as:

1. Determine the appropriate model order for the time-series (Tutorial example: Exploring model order)
2. Fit a model to the data (Tutorial example: Fitting real univariate data)
3. Examine metrics on the model (Tutorial example: MVAR connectivity estimation)
4. Plot results (Tutorial example: Plotting helpers)

The toolbox includes a test suite based around the python py.test module. This test suite runs from the continuous integration environment on the gitlab server and the authors intend to continue expanding the coverage of the test suite.

Screenshots

All screenshots are based around material found in the online tutorials (https://sails.readthedocs.org) in which code to generate them can be found.
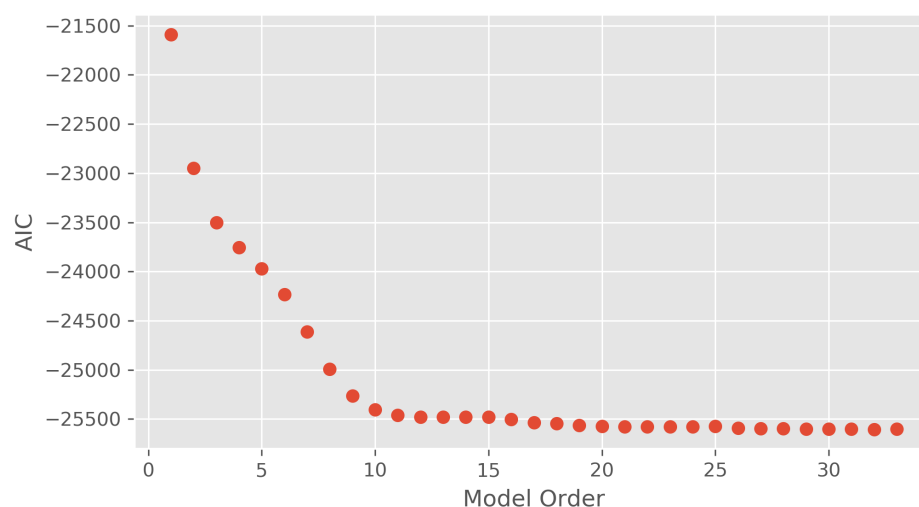

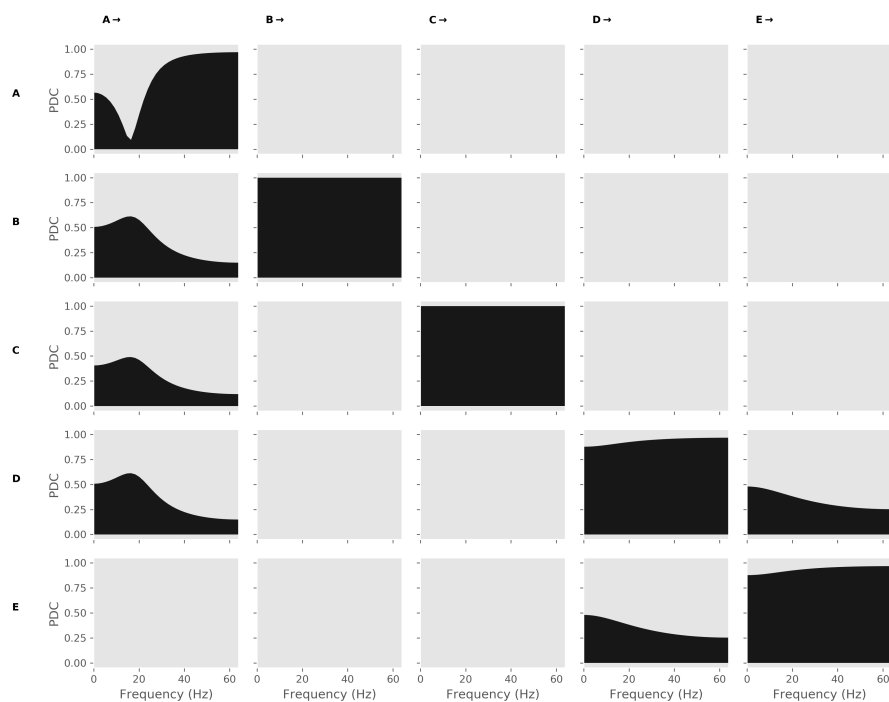**Figure 1:** Example of plot using AIC to determine model order

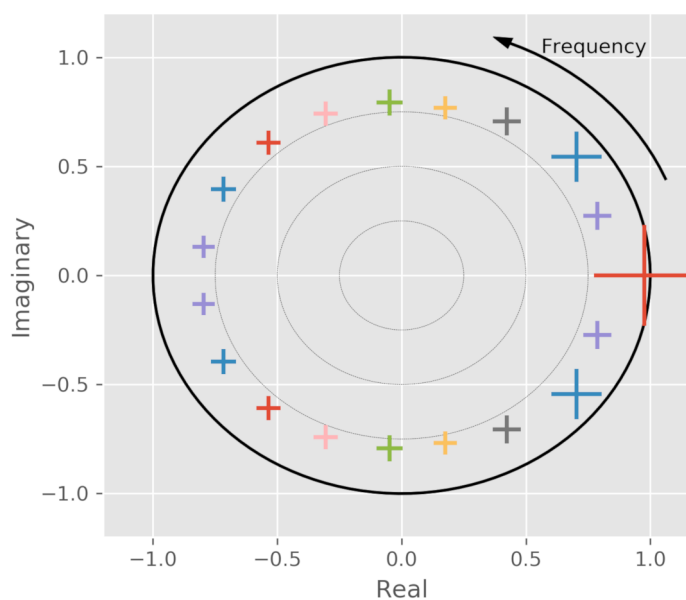**Figure 2:** Example of a Partial Directed Coherence matrix plot

Figure 3: Example of plot showing the pole representation of the AR system

Quinn et al., (2020). SAILS: Spectral Analysis In Linear Systems. *Journal of Open Source Software*, 5(47), 1982. https://doi.org/10.21105/joss.01982
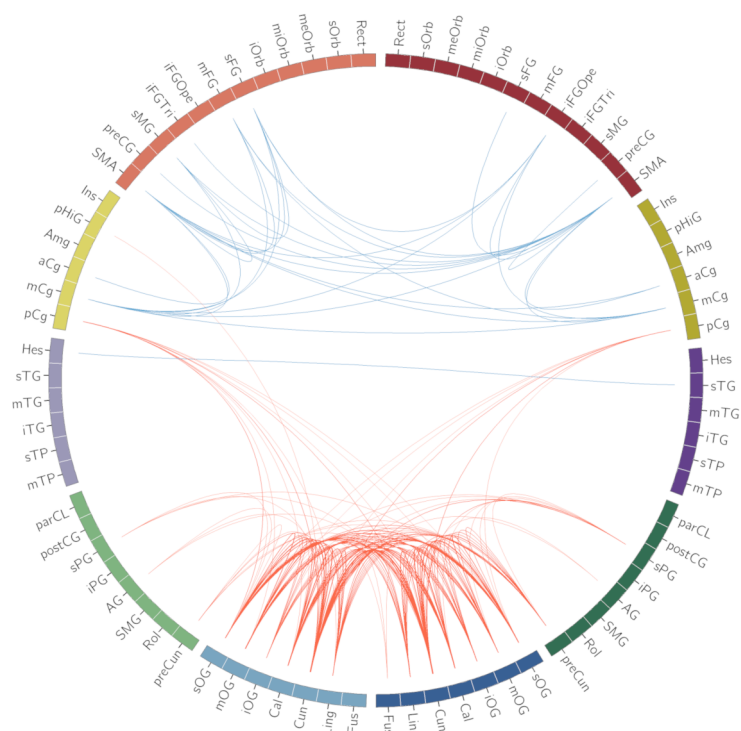
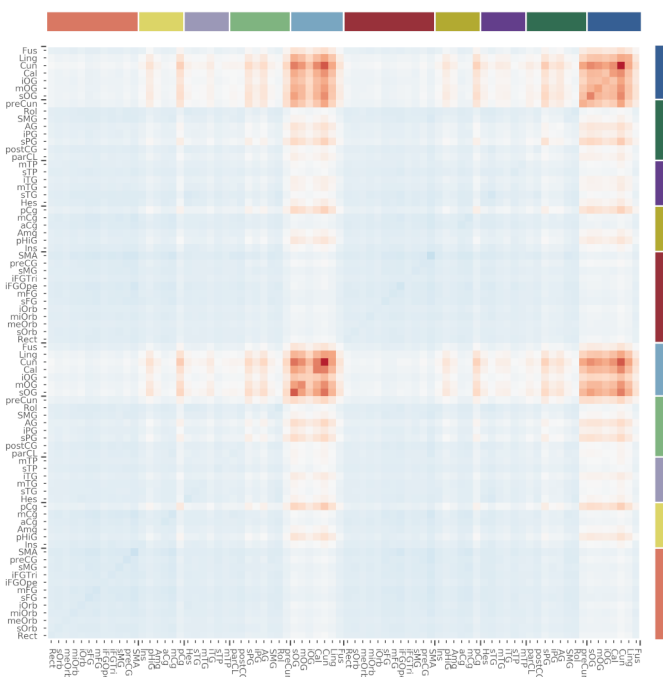**Figure 4:** Example of circular connectivity plot generated using Circos

Figure 5: Example of netmat connectivity plot

References

Baccalá, L. A., & Sameshima, K. (2001). Partial directed coherence: A new concept in neural structure determination. *Biological Cybernetics*, *84*(6), 463–474. https://doi.org/10.1007/pl00007990

Ding, M., Bressler, S. L., Yang, W., & Liang, H. (2000). Short-window spectral analysis of cortical event-related potentials by adaptive multivariate autoregressive modeling: data preprocessing, model validation, and variability assessment. *Biological Cybernetics*, *83*(1), 35–45. https://doi.org/10.1007/s004229900137

Durbin, J., & Watson, G. S. (1950). Testing for Serial Correlation in Least Squares Regression I. *Biometrica*, *37*, 409–428. https://doi.org/10.1093/biomet/37.3-4.409

Kaminski, M. J., & Blinowska, K. J. (1991). A new method of the description of the information flow in the brain structures. *Biol. Cybern.*, *65*(3), 203–210. https://doi.org/10.1007/bf00198091

Krzywinski M, B. I., Schein J. (2009). Circos: an information aesthetic for comparative genomics. *Genome Res*, *19*, 1639–1645. https://doi.org/10.1101/gr.092759.109

Lawrence Marple Jr, S. (1987). *Digital Spectral Analysis: With Applications*. Prentice-Hall. ISBN: 9780132141499

Lütkepohl, H. (2007). *New Introduction to Multiple Time Series Analysis*. Springer. https://doi.org/10.1007/3-540-27752-8

Neumaier, A., & Schneider, T. (2001). Estimation of parameters and eigenmodes of multivariate autoregressive models. *ACM Transactions on Mathematical Software*, *27*(1), 27–57. https://doi.org/10.1145/382043.382304

Pascual-Marqui, R. D., Biscay, R. J., Bosch-Bayard, J., Lehmann, D., Kochi, K., Kinoshita, T., Yamada, N., & Sadato, N. (2014). Assessing direct paths of intracortical causal information flow of oscillatory activity with the isolated effective coherence (iCoh). *Frontiers in Human Neuroscience*, *8*. https://doi.org/10.3389/fnhum.2014.00448