dit: a Python package for discrete information theory

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Summary

`dit`("Dit: A Python Package for Discrete Information Theory. Available at: Https://Github.com/Dit/Dit" n.d.) is a Python package for the study of **d**iscrete **i**nformation **t**heory. Information theory is a mathematical framework for the study of quantifying, compressing, and communicating random variables (Cover and Thomas 2006)(MacKay 2003)(Yeung 2008). More recently, information theory has been utilized within the physical and social sciences to quantify how different components of a system interact. `dit` is primarily concerned with this aspect of the theory.

Information theory is a powerful extension to probability and statistics, quantifying dependencies among arbitrary random variables in a way tha tis consistent and comparable across systems and scales. Information theory was originally developed to quantify how quickly and reliably information could be transmitted across an arbitrary channel. The demands of modern, data-driven science have been coopting and extending these quantities and methods into unknown, multivariate settings where the interpretation and best practices are not known. For example, there are at least four reasonable multivariate generalizations of the mutual information, none of which inherit all the interpretations of the standard bivariate case. Which is best to use is context-dependent. `dit` implements a vast range of multivariate information measures in an effort to allow information practitioners to study how these various measures behave and interact in a variety of contexts. We hope that having all these measures and techniques implemented in one place will allow the development of robust techniques for the automated quantification of dependencies within a system and concrete interpretation of what those dependencies mean.

`dit` implements the vast majority of information measure defined in the literature, including entropies (Shannon(Cover and Thomas 2006), Renyi, Tsallis), multivariate mutual informations (co-information(Bell 2003)(McGill 1954), total correlation(Watanabe 1960), dual total correlation(Te Sun 1980)(Han 1975)(Abdallah and Plumbley 2012), CAEKL mutual information(Chan et al. 2015)), common informations (Gács-Körner(Gács and Körner 1973)(Tyagi, Narayan, and Gupta 2011), Wyner(Wyner 1975)(W. Liu, Xu, and Chen 2010), exact(Kumar, Li, and El Gamal 2014), functional, minimal sufficient statistic), and channel capacity(Cover and Thomas 2006). It includes methods of studying joint distributions including information diagrams, connected informations(Schneidman et al. 2003)(Amari 2001), marginal utility of information(Allen, Stacey, and Bar-Yam 2017), and the complexity profile(Y. Bar-Yam 2004). It also includes several more specialized modules including bounds on the secret key agreement rate(Maurer and Wolf 1997), partial information decomposition(Williams and Beer 2010), rate-distortion theory(Cover and Thomas 2006) & information bottleneck(Tishby, Pereira, and Bialek 2000), and others. Please see the dit homepage for a complete and up-to-date list.

Where possible, the implementations in `dit` support multivariate, conditional forms even if not defined that way in the literature. For example, `dit` implements the multivariate, conditional exact common information even though it was only defined for two variables.

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